Beam and Spin Dynamics in an RF Wien-Filter

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1 Setup of the RF Wien-Filter

For measurements of permanent EDMs (**E**lectric **D**ipole **M**oments) of charged, light hadrons in dedicated storage rings, a technique, which requires a spin tune modulation by an RF Dipole without inducing any coherent beam oscillations, has been proposed [1].
In the course of 2014 and 2015, a prototype RF (**R**adio-**F**requency) Dipole with perpendicular electric and magnetic fields has been successfully commissioned and tested at COSY [2][3]. With careful adjustments of the amplitudes of the radial magnetic and vertical electric field, a field configuration, where the *Lorentz* force onto the revolving particles in the synchrotron ring cancels out, can be achieved. The desired field configuration forms a velocity filter named after its inventor, *Wilhelm Wien* [4].
The magnetic dipole field for the desired *Wien*-Filter field combination is generated by means of a coil wound lengthwise around a ceramic part of the beam-pipe. Ferrite blocks bundle the field lines and flatten the transverse field distribution. To generate an oscillating field, the coil is connected to an adjustable, parallel resonance circuit with a quality factor of $Q \approx 20$. A similar, but separate resonance circuit drives the electric RF Dipole. The electric field is generated by the potential difference between two stainless steel electrodes inside the vacuum chamber, spanned over glass rods held by a frame inside the flanges of the ceramic beam-chamber. For details see Fig.1.

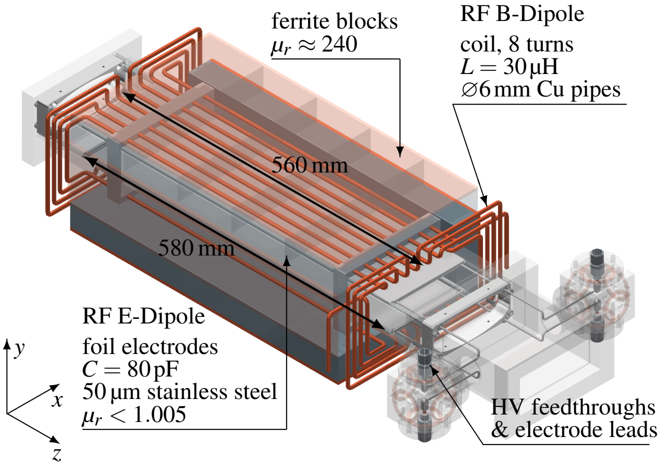

Fig. 1: A view inside the RF ExB-Dipole.

The coil itself is water cooled and fans dissipate the remaining lost power. Therefore, it is possible to run the system up to 90 W input power in continuous, long term operation. The corresponding operating parameters have been collected in Table 1.

	RF B-Dipole	RF E-Dipole
$\hat{U}$		2 kV
$\int \hat{E}_y \, dl$		24.1 kV
$\hat{I}$	5 A	
$\int \hat{B}_x \, dl$	0.175 T mm	
f_{RF} range	630 kHz to 1170 kHz	630 kHz to 1060 kHz

Table 1: The RF ExB-Dipole at 90 W RMS input power.

2 Beam Dynamics

Figs. 2a and 2b show that the transverse distribution of the main field components is flat across the center of the beam chamber. But due to different drop-off rates of the electric and magnetic field, particles will encounter a down-up kick at the entrance and a corresponding up-down kick at the exit of the *Wien*-Filter. The geometry has been optimized insofar that particles with the reference momentum of 970 Mev/c, entering the system on axis, won't get any vertical excursion, as shown in Fig. 2c. Particles off momentum will probe different *Lorentz* forces, leading to a slight spread in vertical beam size in the order of 20 nm for an energy spread of $\Delta\gamma/\gamma = 10^{-4}$. The main field asymmetry stems from the feeds to the coil and the electrodes. They give a small horizontal kick, which steers the particles ≈ -40 nm off axis 1 m behind the device center.
For comparison, Fig: 3 shows the phase space distribution of particles at the location of the RF Wien-Filter, calculated using the beam optics functions from the September 2014 and May 2015 JEDI beam-times. Considering a normalized

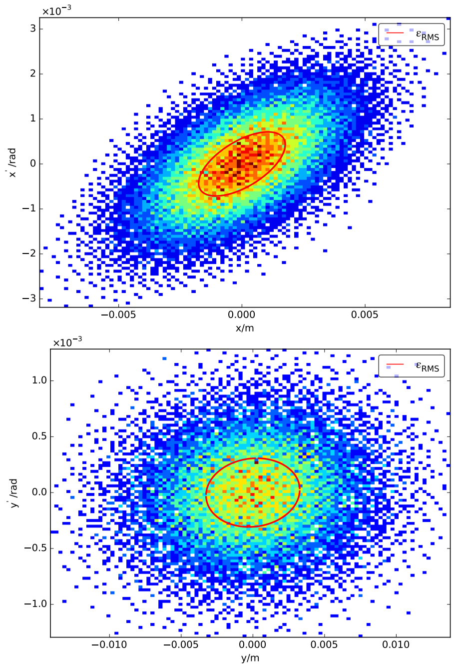

Fig. 3: Horizontal and vertical phase-space distribution in the center of the RF Wien-Filter with the RMS emittance ellipse.

R.M.S. emittance of $\varepsilon \approx 1\,\pi$ mm mrad, which is typical for a well cooled deuteron beam at COSY, one gets an estimate for the beam size at the location of the RF ExB-Dipole.

$$\beta_x = 4.1\,\text{m} \Rightarrow \langle x \rangle = \sqrt{\varepsilon\beta_x} = 2.0\,\text{mm} \quad (1)$$

$$\beta_y = 21.1\,\text{m} \Rightarrow \langle y \rangle = \sqrt{\varepsilon\beta_y} = 3.9\,\text{mm}. \quad (2)$$

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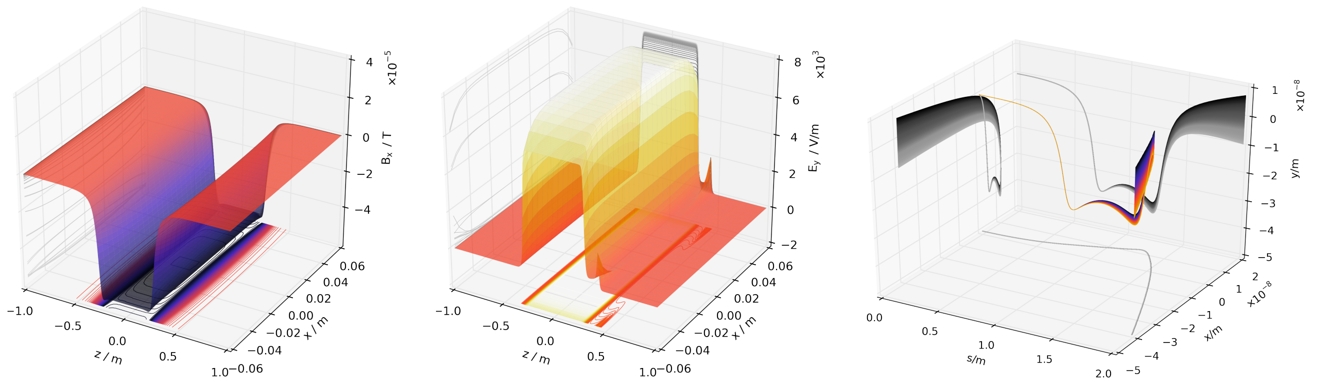


(a) $\hat{B}_x$ across the center of the beam-pipe. (b) $\hat{E}_y$ across the center of the beam-pipe. (c) Trajectories for deuterons passing through the ExB field distribution.

Fig. 2: Simulation of the main field components generated by the RF ExB-Dipole, normalized to an RF B current amplitude of 1 A. The right panel shows the result of particle tracking through these fields. The Deuterons have been initialized on the reference orbit ($x_0 = y_0 = 0\,\mathrm{mm}$, $x'_0 = y'_0 = 0\,\mathrm{rad}$).

Since the field strengths are small in the first place and the *Lorentz* force onto the particles is canceled out in average, the beam disturbance of the RF *Wien*-Filter is 5 orders of magnitude below these beam size estimates.

3 Spin Dynamics

The system has been tested with spin manipulation on a vertically polarized deuteron beam. Plugging the *Lorentz* force cancellation into the *Thomas-BMT* Equation [5] yields a simple formula for the spin precession frequency $\vec{\Omega}$ in an ideal *Wien*-Filter [6].

$$\vec{F}_{\mathrm{L}} = q\left(\vec{E} + c\vec{\beta}\times\vec{B}\right) \stackrel{!}{=} 0 \quad \Rightarrow \quad \vec{\Omega}_{\mathrm{WF}} = \frac{q}{\gamma m}\frac{1+G}{\gamma}\vec{B} \quad (3)$$

The particles sample the localized RF field distribution (see Figs. 2a,2b) once every turn n. Its contribution can be approximated by the integrated field along the particles path assigned to a point-like device at an orbital angle θ:

$$b(\theta) = \int \hat{B}_x \mathrm{d}l \cos\left(\frac{f_{\mathrm{RF}}}{f_{\mathrm{rev}}}\theta + \phi\right) \sum_{n=-\infty}^{\infty} \delta(\theta - 2\pi n). \quad (4)$$

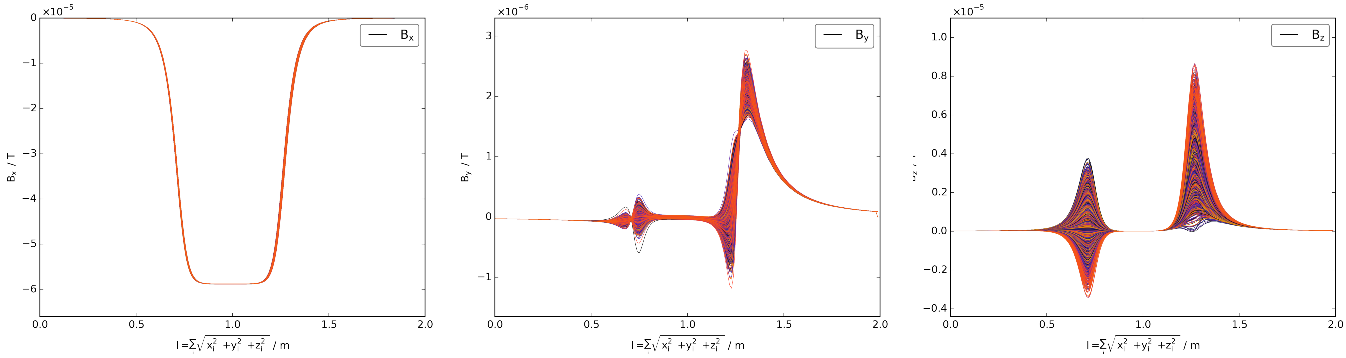
The field integrals of a deuteron beam as used during the JEDI beam-times are shown in Fig. 4. For a realistic beam with momentum and velocity spread as shown in Fig. 3, each particle will sample slightly different field strengths but, for the main field components $\hat{B}_x$ and $\hat{E}_y$, the resulting distribution of the field integrals is very narrow with a standard deviation three orders of magnitude of its mean value. The particles spread out across the phase space will mainly pick up different fringe field components. Since the unwanted field components are small, two orders of magnitude below the main field in case if the magnetic field and tree orders of magnitude below for the main electric field component, equation 4 is a good approximation for the determination of the spin motion in the RF *Wien-Filter*.
The resonance strength ε of this field configuration is given by the amount of spin rotation inside the device per turn. In the absence of RF induced coherent beam beam motion, it solely depends on the strength of the RF field. In this case, it can be calculated by the *Fourier* integral for spin kicks in many consecutive turns [7, 8]:

$$\begin{aligned}\varepsilon_K &= \frac{\Omega_{\mathrm{WF}}}{\Omega_{\mathrm{rev}}} = \frac{1+G}{2\pi\gamma}\oint \frac{b(\theta)}{B\rho} e^{iK\theta}\,\mathrm{d}\theta \\ &= \frac{1+G}{4\pi\gamma}\frac{\int \hat{B}_x \mathrm{d}l}{B\rho}\sum_n e^{\pm i\phi}\delta\left(n - K \mp \frac{f_{\mathrm{RF}}}{f_{\mathrm{rev}}}\right). \quad (5)\end{aligned}$$

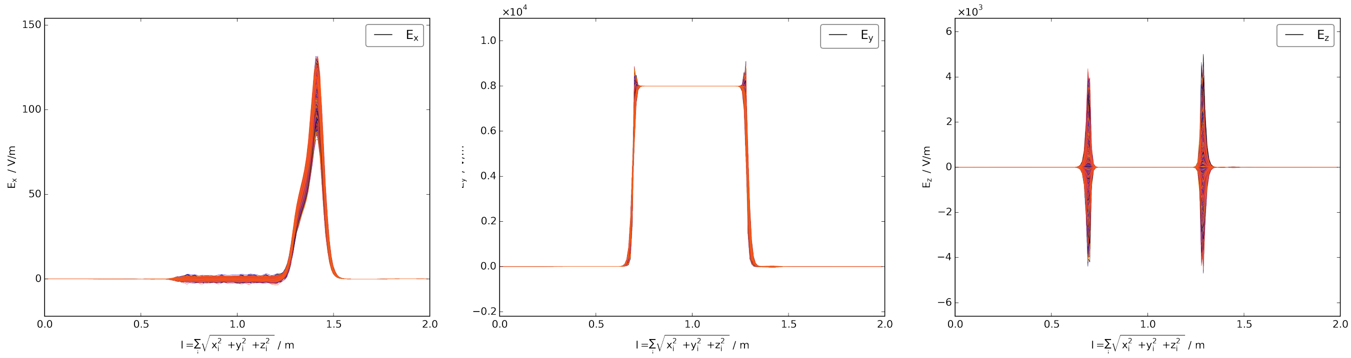
If the RF *Wien*-Filter is operated on a harmonic of the spin tune in the storage ring, the spin kicks induced by the radial magnetic field add up and introduce an additional, continuously rotation of the particles' spins around the magnetic field's axis. For deuterons with a momentum of $970\,\mathrm{MeV/c}$ the spin tune is $\nu_S \approx \gamma G = -0.1609$. This gives resonance side-bands at

$$f_{\mathrm{RF}} = f_{\mathrm{rev}}|n - \gamma G| = \begin{cases} 630\,\mathrm{kHz} \text{ for } n = 1 \\ 871\,\mathrm{kHz} \text{ for } n = -1 \end{cases} \quad (6)$$

Both are available for studies inside the frequency range of the RF *Wien*-Filter. Resonance strength measurements have been performed during the September 2014 and May 2015 JEDI beam-times at COSY. The driven spin rotation was detected by observing the vertical component of a polarized deuteron beam as an left-right asymmetry in the angular distribution in $^{12}\mathrm{C}(\vec{\mathrm{d}},\mathrm{d})$ [9]. The resulting oscillation frequency of the rate-asymmetry in the four-quadrant polarimeter detector was measured[10] and is directly proportional to the resonance strength in Eq. 5.
Comparison measurements were performed between the RF ExB-Dipole in *Wien*-Filter-mode, the RF ExB-Dipole operated without compensating electric field as a pure RF B-Dipole and with only the electric field switched on as a pure RF E-Dipole and with an already installed RF Solenoid. By modifying the strengths of two of the main quadrupole families in the synchrotron, the fractional vertical betatron tune q_y is shifted and the betatron oscillation frequency is moved farther or closer to the frequency of the RF systems, thereby varying the degree of induced coherent beam oscillations.
Fig. 5 shows, that, as for an RF Solenoid, the RF *Wien*-Filter resonance strength is independent of the vertical betatron tune. In contrast, the resonance strength of the pure RF dipoles is dominated by the interference between the driven spin motion and the one induced by coherent beam oscillations. Experimentally, this effect was already observed by



(a) $\int \hat{B}_x \mathrm{d}l = -3.479 \cdot 10^{-5}\,\mathrm{T\,m}$, $\sigma_{\int \hat{B}_x \mathrm{d}l} = 0.005 \cdot 10^{-5}\,\mathrm{T\,m}$
(b) $\int \hat{B}_y \mathrm{d}l = 4.14 \cdot 10^{-7}\,\mathrm{T\,m}$, $\sigma_{\int \hat{B}_y \mathrm{d}l} = 0.07 \cdot 10^{-7}\,\mathrm{T\,m}$
(c) $\int \hat{B}_z \mathrm{d}l = 7.2 \cdot 10^{-7}\,\mathrm{T\,m}$, $\sigma_{\int \hat{B}_z \mathrm{d}l} = 0.4 \cdot 10^{-7}\,\mathrm{T\,m}$



(d) $\int \hat{E}_x \mathrm{d}l = 13.0\,\mathrm{V}$, $\sigma_{\int \hat{E}_x \mathrm{d}l} = 0.6\,\mathrm{V}$
(e) $\int \hat{E}_y \mathrm{d}l = 4793\,\mathrm{V}$, $\sigma_{\int \hat{E}_y \mathrm{d}l} = 7\,\mathrm{V}$
(f) $\int \hat{E}_z \mathrm{d}l = 0.002\,\mathrm{V}$, $\sigma_{\int \hat{E}_z \mathrm{d}l} = 2.000\,\mathrm{V}$

Fig. 4: Amplitudes of the electromagnetic field components of the RF *Wien*-Filter, evaluated along each particles trajectory for a beam set up as shown in Fig. 3. The captions show the mean of all individually integrated trajectories and its standard deviation.

experiments with resonance strength measurements by the SPIN@COSY Collaboration [11]. A constant fit to the mea-

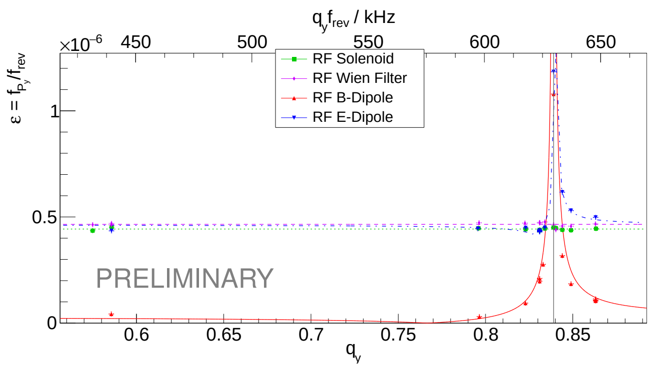


Fig. 5: Results of the resonance scans at the nominal tune of COSY at $q_y = 0.56$ and for a set of vertical betatron tunes with sideband frequencies around the spin resonance at, depicted by the vertical line at 629 777.3 Hz.

surements of the RF *Wien*-Filter gives a resonance strength of $\varepsilon = (4.65 \pm 0.04) \cdot 10^{-7}$ for the measured current amplitude of $\hat{I} = (0.740 \pm 0.005)\,\mathrm{A}$. The resulting normalized effective field strength of the system agrees well with the estimate from the field simulations (see figure 4)

$$\frac{\int \hat{B}_x \mathrm{d}l}{\hat{I}} = \varepsilon \frac{4\pi\gamma}{1+G} \frac{B\rho}{\hat{I}} = (3.40 \pm 0.05) \cdot 10^{-5}\,\mathrm{Tm/A}. \quad (7)$$

4 Conclusion

As a preparation for future EDM experiments in storage rings, a first prototype of an RF *Wien*-Filter has been commissioned at COSY. We have shown, that this device generates a configuration of RF dipole fields which allow spin manipulation in a storage ring without beam disturbance.

References:

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